Reviewer #1: I am very sympathetic to the idea that the Hilbert space is not adequate. And this manuscript offers serious reasons for this. However, I think that some of these arguments can be improved or, if not, avoided, and so I recommend that the authors make the following changes. It would be a shame if good arguments are ignored because a reader finds fault in the weaker arguments.

We thank the reviewer for their sympathetic view that Hilbert spaces are not adequate.  
  
1) About page 1, second column, lines 40-58:  
I don't think there is a law saying that observers should be able to know everything about the physical universe, so if most of the data in the state vector can't be accessed by us, this doesn't make the state vector unphysical. Anyway, in quantum mechanics, the string of bits of information that we can extract by measurements is not supposed to identify any vector in the state space. For example, quantum measurements don't give the state of a qubit prior to the measurement, it only gives one of the two eigenvalues. From a qubit we get a bit, even if there are infinitely many available states for the qubit.  
  
I recommend that the authors either find a better argument that it is necessary for the observer to be able to access anything about the quantum state, or to avoid this.  
  
We have considerably expanded the discussion of what we mean by “physicality” in the updated manuscript and hope that it is now clearer. We also chose to add text that addresses some objections by reviewers from previous journal submissions. We have clarified in the text that quantum states—and states more generally in physics—describe preparations rather than measurements, and the preparation of a pure state would fully specify the state vector, e.g. a spin-1/2 particle can be prepared in a particular direction.

2) Subsection IIB. Completeness  
  
An argument in this subsection is the reverse of (1): just like there is no reason to believe that we can access everything, there is no reason to believe that we should have full control of the state vector, let alone that we should be able to prepare any state vector. So I recommend the same treatment that I recommended for (1).

In our mind, what differentiates science from pure mathematics is that it is evidence-based, empirical. The objects within the domain of physics must therefore, often with some idealization, have a clear connection to experimental practice. We hope that the expanded section “On Physicality” helps clarify our view of this.  
  
3) Regarding the appendix section V, the explanations between Propositions 24 and 31. The authors point out that there are one-parameter groups of unitary evolution operators that lead to oscillations of expectation values from finite to infinite. But in reality there is only one Hamiltonian, and therefore a one-parameter group of unitary evolution operators. So I think this argument doesn't hold, because it is based on a group specifically constructed to be problematic (like a straw man). And I point this out to the authors despite their preemptive assessment of possible objections as "missing the point", because I think this is a fallacy. There is only one evolution law in a quantum universe. The authors also wrote that you can use certain unitary transformations to find bases in which finite expectation values become infinite, but this would be akin to claiming to prove the physical inadequacy of differential geometry because you can find singular coordinate systems.  
  
As noted in the appendix, it is possible to write down a Hamiltonian for these unphysical unitary transformations. In the world of high-energy theory looking for new physics beyond the Standard Model, researchers routinely try to create new Hamiltonians (or more frequently new Lagrangians) that they believe might possibly describe so-far-undiscovered new forces and interactions. While that is in the context of quantum field theory rather than nonrelativistic quantum mechanics, the risk to the efficient progress of research of unmindfully allowing unphysical objects in the mathematics is similar. To look to a completely different field, within the hierarchy of safety controls, elimination, substitution, and engineering controls are strongly preferred over administrative controls to manage hazards—putting up a fence at the edge of a cliff is much safer than simply putting a sign saying, “Beware of cliff.” By using Hilbert spaces for physical calculations in quantum mechanics, the community is effectively relying on “administrative controls” to avoid incorrect results, i.e. considering if a mathematically allowed calculation makes physical sense and discarding it if not. For physics problems in realms close to everyday human experience, our physical intuition is very reliable. However, for problems within e.g. quantum physics, quantum field theory, or cosmology, our physical intuition is much less powerful, and in fact these areas of physics rely much more heavily on the mathematics. Our paper is effectively advocating that we try to implement “engineering controls” to ensure the soundness of calculations in quantum mechanics—if we can find a suitable mathematical space that does not allow unphysical objects in the first place, then there is no risk of accidentally failing to discard unphysical results. We have added text clarifying this argument.

Regarding differential geometry, the notion of a differentiable manifold is exactly excluding those coordinate systems that would introduce singularities (i.e. nondifferentiable coordinates). Therefore yes, topologically continuous transformations that are not differentiable exist, but they are excluded from the atlas as they break the differentiable structure. What we are arguing is that mathematical spaces used in physics should already come equipped with the proper structure that excludes physically pathological behavior. Note that the structure of Schwartz spaces comes equipped with a notion of differentiability, as transformations that map Schwartz functions to Schwartz functions preserve smoothness. In quantum mechanics, this is enough to prevent transformations that map finite expectation values to infinite ones.

4) I would like to see a more serious discussion of the rigged Hilbert spaces. There is a good reason why the first space in the Gel'fand triple can be a Schwarz space, see refs [R1] and Section IIB in ref. [R2] below. I think the authors should not reject rigged Hilbert spaces by simply saying  
  
"Therefore, a rigged Hilbert space is physical only if all three spaces are physical. If the Hilbert space is not physical, neither is the rigged Hilbert space."  
  
This assumes that all the three spaces in the Gel'fand triple are supposed to represent physical states, which is not the case. So the rejection of rigged Hilbert spaces can't be simply reduced to the rejection of Hilbert spaces as representing physical states, which is argued in the rest of the paper. Rather, I suggest they discuss the physical meanings attributed to the Gel'fand triple from the literature, as in refs [R1,R2] below, and in the references cited in the manuscript, in particular [27], which give more nuanced physical interpretations. As it is now, Sec. VI only introduces the rigged Hilbert space and gives a vague motivation for them, while in fact the researchers who proposed it and advocate for its use in Quantum Mechanics made a better case for this than it appears in this manuscript.  
  
We have modified the discussion of rigged Hilbert spaces, describing more explicitly their motivation and how they do not address the issues that we are raising in our paper, and in fact, in our view, further mathematically obscure important physical differences. We are unsure what part of reference [R1] the reviewer has in mind, as it does not appear to us to have a clear physical mapping of the mathematical objects. The words “physical” and “state” each appear just once in the 22-page paper, and in the Introduction, it is stated, “the actual interpretation in terms of labelled observables is irrelevant, and it is sufficient to have a suitable set of distinguished operators on Hilbert space, which are to be made continuous.” The Conclusion of reference [R2] instead makes clearer statements about the motivation for rigged Hilbert spaces, and we have added it as a reference.

5) I endorse the arguments for the Schwartz spaces, especially as presented in VII. I think this is the most interesting part of the manuscript, and I recommend it to be promoted from an appendix section to a regular section of the manuscript, or to expand section III.

We also find the details of the Schwartz spaces very interesting, and that is why we included them. However, the core result of this particular work is just that the completeness of Hilbert spaces introduces unphysical objects within the mathematics. A proper solution would require identifying a suitable alternative for completeness. This would require a broader discussion than just Schwartz spaces, which are simply taken as an example to illustrate that better options may exist.

For example, we found a notion of quasi-completeness, discussed here: <https://www-users.cse.umn.edu/~garrett/m/fun/notes_2012-13/07d_quasi-completeness.pdf>. We exchanged email with the author, who told us that he is not aware of any published work on this, so we do not mention it in the manuscript.

We are not proposing Schwartz spaces as a definitive answer, and in fact in the manuscript we point out several issues that it is currently unclear to us how to address with Schwartz spaces. We feel that including a lengthy section on Schwartz spaces in the body of the paper would distract from the main point.

A minor remark: the references are cited in an unusual way.[1]

We will change the format of the citations to conform to the journal standard, should the article be accepted for publication.

I recommend that, to be published, the authors should make these changes, particularly a considerably much more serious discussion of the physical motivations given in the literature for the rigged Hilbert spaces.  
  
We hope that we have made clearer the different problems with Hilbert spaces that rigged Hilbert spaces seek to solve and that we highlight in our paper.

References:  
[R1] Roberts, J. E. "Rigged Hilbert spaces in quantum mechanics." Communications in Mathematical Physics 3 (1966): 98-119, <https://link.springer.com/content/pdf/10.1007/BF01645448.pdf>  
[R2] Celeghini, Enrico, M. Gadella, and M. A. Del Olmo. "Applications of rigged Hilbert spaces in quantum mechanics and signal processing." Journal of Mathematical Physics 57.7 (2016)., <https://mathphys.uva.es/files/2016/08/rigged.pdf>